P. 305 no: 49

a) $\frac{\sin\theta \cot^2\theta}{\cos\theta} = \cot\theta$

$\frac{\sin\theta}{\cos\theta} \cdot \frac{\cos^2\theta}{\sin^2\theta} = \frac{\cos\theta}{\sin\theta} = \cot\theta$

b) $\sin x(1+\tan x) + \cos x(1+\cot x) = \sec x + \operatorname{cosec} x$

$\sin x\left(1+\frac{\sin x}{\cos x}\right) + \cos x\left(1+\frac{\cos x}{\sin x}\right)$

$\sin x + \frac{\sin^2 x}{\cos x} + \cos x + \frac{\cos^2 x}{\sin x}$

$\frac{\sin^2 x}{\cos x} + \cos x + \frac{\cos^2 x}{\sin x} + \sin x$

$\frac{\sin^2 x + \cos^2 x}{\cos x} + \frac{\cos^2 x + \sin^2 x}{\sin x} = \frac{1}{\cos x} + \frac{1}{\sin x} = \sec x + \operatorname{cosec} x$

c) $\frac{2\sin x\cos x - \cos x}{1 - \sin x + \sin^2 x - \cos^2 x} = \frac{1}{\tan x}$

$\frac{\cos x(2\sin x - 1)}{-\sin x + \sin^2 x + 1 - \cos^2 x}$

$\frac{\cos x(2\sin x - 1)}{-\sin x + \sin^3 x + \sin^2 x} = \frac{\cos x(2\sin x - 1)}{-\sin x + 2\sin^2 x} = \frac{\cos x(2\sin x - 1)}{\sin x(-1 + 2\sin x)}$

$= \frac{\cos x}{\sin x} = \frac{1}{\tan x}$

d) $\cos x(\tan x + 2)(2\tan x + 1) = 2\sec x + 5\sin x$

$\cos x\left(\frac{\sin x}{\cos x} + 2\right)\left(2\frac{\sin x}{\cos x} + 1\right) = (\sin x + 2\cos x)\left(2\frac{\sin x}{\cos x} + 1\right)$

$= \frac{2\sin^2 x}{\cos x} + \sin x + 4\sin x + 2\cos x$

$= \frac{2\sin^2 x}{\cos x} + 2\cos x + 5\sin x$

$= \frac{2\sin^2 x + 2\cos^2 x}{\cos x} + 5\sin x$

$= \frac{2(\sin^2 x + \cos^2 x)}{\cos x} + 5\sin x$

$= \frac{2}{\cos x} + 5\sin x = 2\sec x + 5\sin x$

f) $\frac{\cos x + \sin x}{\sec x + \operatorname{cosec} x} = \cos x \cdot \sin x$

$\frac{\cos x + \sin x}{\frac{1}{\cos x} + \frac{1}{\sin x}} = \frac{\cos x + \sin x}{\frac{\sin x + \cos x}{\cos x \cdot \sin x}} = (\cos x + \sin x)\left(\frac{\cos x \cdot \sin x}{\sin x + \cos x}\right)$

$= \cos x \cdot \sin x$

g) $\frac{1}{1-\sin x} + \frac{1}{1+\sin x} = 2\sec^2 x$

$\frac{(1+\sin x)+(1-\sin x)}{(1+\sin x)(1+\sin x)} = \frac{2}{1-\sin^2 x} = \frac{2}{\cos^2 x} = 2\sec^2 x$

h) $\frac{\tan x}{\sec x - 1} + \frac{\tan x}{\sec x + 1} = 2\operatorname{cosec} x$

$\frac{\tan x(\sec x+1) + \tan x(\sec x - 1)}{\sec^2 x - 1}$

$\frac{\tan x\,(\sec x + 1 + \sec x - 1)}{\tan^2 x}$

$\frac{\tan x\,(2\sec x)}{\tan^2 x} = \frac{2\sec x}{\tan x}$

$= \frac{2\,\frac{1}{\cos x}}{\frac{\sin x}{\cos x}} = 2\,\frac{1}{\sin x} = 2\operatorname{cosec} x$

i) $(1+\sin x+\cos x)^2 = 2(1-\sin x)(1+\cos x)$

$1 - \sin x + \cos x - \sin x + \sin^2 x - \sin x\cos x + \cos x - \sin x\cos x + \cos^2 x$

$1 - 2\sin x + 2\cos x - 2\sin x\cos x + 1$

$2 - 2\sin x + 2\cos x - 2\sin x\cos x$

$2(1 - \sin x + \cos x - \sin x\cos x)$

$2(1 + \sin x + \cos x(1 - \sin x))$

$2((1-\sin x)(1+\cos x))$

j) $\frac{\sin x + \tan x}{\operatorname{cosec} x + \cot x} = \sin\theta \cdot \tan\theta$ ou $\sin\theta \cdot \frac{\sin\theta}{\cos\theta}$

$\frac{\sin x + \frac{\sin x}{\cos x}}{\frac{1}{\sin x} + \frac{\cos x}{\sin x}} = \frac{\frac{\sin x\cdot\cos x + \sin x}{\cos x}}{\frac{1+\cos x}{\sin x}}$

$= \frac{\sin x(1+\cos x)}{\cos x} \cdot \frac{\sin x}{(1+\cos x)}$

$= \frac{\sin x}{\cos x} \cdot \sin x$

$= \tan x \cdot \sin x$